

# FIELD THEORY ANALYSIS OF CONICALLY SHAPED COUPLING ELEMENTS IN DUAL MODE FILTERS AND POLARIZERS

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**Abstract** - The S-parameters of conically shaped coupling elements in dual mode filters and polarizers in circular waveguides have been derived using the mode matching method. The coupling elements are single or double ridges of uniform angular thickness placed at any arbitrary angle to the excitation in order to couple the orthogonal polarizations. Using ridges of this shape enables the analysis entirely in cylindrical coordinates and the evaluation of some of the coupling integrals analytically. A 90° differential phase shift between the orthogonal modes for a polarizer application has also been realized using a double ridged circular waveguide.

Polarizers can be designed as well from this analysis. While a single ridge is used in dual mode filters, a polarizer uses two [5,6] placed at 45° to the excitation. Once again, for a polarizer, the two flats can also be replaced with conically shaped ridges. A transformer approach to the design of polarizers using a quadruple ridged circular waveguide was presented in [7]. Here, a double ridged transformer approach to design a polarizer will be presented. In other words a 90° differential phase shift between the orthogonally polarized inputs has been achieved using a double ridged circular waveguide where the ridges are positioned parallel to one excitation and perpendicular to the other.

## I Introduction

Dual mode waveguide filters and polarizers are widely used in satellite applications [1]. A rigorous design of dual mode filters was presented in [2,3]. The analysis of coupling screws was performed using a combined mode-matching finite element analysis in these references. A combination of methods becomes essential when the coupling screw in a circular waveguide is shaped as a rectangular post. However, by conically shaping the ridges, the analysis can be performed entirely within the framework of the mode matching method, instead of a space discretization approach or a combined mode matching space discretization approach. The analysis of symmetric double ridged circular waveguide discontinuities for the design of evanescent mode filters and transformers has been presented in [9]. This paper describes the computation of the generalized scattering matrices of a discontinuity from empty circular waveguide to a single and double ridged circular waveguides, where the ridges are placed at an arbitrary angle to the excitation. This is the first time S-parameters of conically shaped ridges for orthogonal mode coupling have been calculated on the basis of a rigorous description of the electromagnetic fields. The resulting algorithm is very fast and suitable for CAD of dual mode filters and polarizers.

## II Theory

The full-wave mode matching approach to an asymmetric discontinuity is presented below. The discontinuity from a circular waveguide to a single ridged or a double ridged waveguide is shown in Figure 1(a-d). The ridges are positioned at an arbitrary angle to the excitation. The solution to Helmholtz equation in cylindrical coordinates is written as a sum of incident and reflected modes propagating in the  $z$  direction with unknown amplitude coefficients. At such a discontinuity, since there exists no symmetry, both the polarizations of the fundamental and higher order  $TE_{n,m}$  and  $TM_{n,m}$  modes get excited. The electric and magnetic potential functions in the circular waveguide (region I) can be written as a sum of the two components of the orthogonal polarizations [4]. The coefficients of these potential functions are chosen as the power normalization constants and are obtained by setting the magnitude of power carried in each mode to unity [6].

At the region II of the discontinuity which is either a single or a double ridged circular waveguide, the potential functions are written as a sum of the potential functions in subregions 1 and 2 and include modes of both the polarizations as well. The procedure to evaluate the eigenvalues

of a single and double ridged circular waveguide has been presented in [8]. In the eigenvalue analysis the ridge was positioned at  $\phi = 0$  (Figure 1(e) and 1(f)) and electric and magnetic wall symmetries (ew/mw) were used to determine the eigenvalues of the orthogonal polarizations of all the modes of the structures. The inclination of the single ridge to the  $\phi = 0$  line is denoted by  $\alpha$  in the present analysis. This angular displacement does not change the eigenvalues of the structure. However, the potential functions should incorporate the change in the boundary condition due to rotation of the ridge. This means that the lines of symmetry also get rotated by  $\alpha$ . For a single ridged circular waveguide the magnetic potential functions in the subregions 1 and 2 are given below.

$$\begin{aligned}\psi^{(1h)} &= \sum_{p=1}^R \sum_{n=r}^{N1} A_n J_n(k_{c_p}^{IIh} \rho) \\ &\quad \cdot \begin{cases} \sin n(\phi - \alpha) & r = 1 \text{ for mw} \\ \cos n(\phi - \alpha) & r = 0 \text{ for ew} \end{cases} \\ \psi^{(2h)} &= \sum_{p=1}^R \sum_{m=r}^{N2} C_m [N'_l(k_{c_p}^{IIh} b) J_l(k_{c_p}^{IIh} \rho) - \\ &\quad J'_l(k_{c_p}^{IIh} b) N_l(k_{c_p}^{IIh} \rho)] \\ &\quad \cdot \begin{cases} \cos l(\phi - \theta - \alpha) & r = 1, 3 \text{ and } l = \frac{m\pi}{2(\pi-\theta)} \text{ for mw} \\ \cos l(\phi - \theta - \alpha) & r = 0 \text{ and } l = \frac{m\pi}{\pi-\theta} \text{ for ew} \end{cases}\end{aligned}$$

The amplitude coefficients of the potential functions are determined as explained in [8,9] and power normalized so that the magnitude of power carried in each mode is unity. Similarly the electric potential functions are written as well. For the double ridged case, four combinations of the two lines of symmetry are used to determine the eigenvalues of modes with both the polarizations in a double ridged circular waveguide. The potential functions in the two subregions in this case are similar to that in a single ridged waveguide. From the potential functions, the electric and magnetic fields at the discontinuity are derived. Matching the tangential components of the electric and magnetic fields and using the orthogonality of modes, equations relating the unknown amplitudes of the incident and reflected waves are obtained.

Since the orthogonal polarizations of the incident and reflected waves are included in the analysis, the equations for such a discontinuity also provide the coupling between the orthogonal polarizations. All the matrix equations derived from the matching conditions can be rearranged and inverted to obtain the generalized scattering matrix of a discontinuity from an empty circular to a ridged circular waveguide. By cascading the generalized scattering matrices the discontinuity from an empty circular to a single or double ridged waveguide of finite length is obtained.

For  $\alpha = 0$  the coupling matrices relating the polarizations orthogonal to each other in region *I* and *II* become zero as the discontinuity becomes symmetric. The discontinuity from empty circular waveguide to double ridged circular waveguide of finite length has been computed using this algorithm by setting  $\alpha = 0$ . The analysis of this discontinuity including both the polarizations has been used to design a differential phase shifter.

### III Results

The analysis of an asymmetric discontinuity involves both the polarizations. Hence, the S-matrix of the fundamental orthogonal polarization extracted from the generalized scattering matrix is a  $4 \times 4$  matrix. The sine polarization is the one with the  $E_\rho$  field component varying sinusoidally with  $\phi$  and the cosine polarization is orthogonal to this. In the following discussions and figures, the subscripts 'sc' and 'cs' of the S-parameters denote the coupling from sine to cosine and cosine to sine polarizations, respectively. The subscripts 's' and 'c' denote the coupling from sine to sine and cosine to cosine polarizations, respectively. For a discontinuity with a finitely long single or double ridge in an empty circular waveguide,  $S_{12sc}$  and  $S_{12cs}$  are equal and similarly the  $S_{11sc}$  and  $S_{11cs}$  due to the symmetry property of the S-matrices.

The convergence of the S-parameters of the asymmetric discontinuities discussed here occurs when 70 modes of both the polarizations of  $TE_{n,m}$  and  $TM_{n,m}$  modes are included. Figures 2 and 3 show the variation of the S-parameters of the single and double ridge discontinuities with frequency. For  $\alpha = 45^\circ$  the discontinuity looks alike for both sine and cosine polarization. Hence the  $S_{11c}$  and  $S_{11s}$  are equal and similarly  $S_{12c}$  and  $S_{12s}$  are also equal. The computed values are very close to each other as seen in these figures. It can be logically concluded that maximum coupling is obtained for  $\alpha = 45^\circ$ . This is shown in Figure 4 for a single ridged discontinuity. Also, from this figure it can be observed that because of rotational symmetry of the discontinuity,  $S_{12sc}$  at  $\alpha = 30^\circ$  is equal to that at  $\alpha = 60^\circ$ . Similarly  $S_{12sc}$  at  $\alpha = 15^\circ$  is equal to that at  $\alpha = 75^\circ$ . When the depth of penetration of the ridge increases the parameter  $S_{12sc}$  should increase because the coupling between orthogonal polarizations is increased. This is shown in Figure 5 for a single ridged discontinuity.

The discontinuity from empty circular waveguide to a double ridged circular waveguide has been computed for the setting  $\alpha = 0$ . The resulting S-parameters of the fundamental mode (cosine polarized) with the present algorithm was verified with the theory and measurements presented in [9]. The S-parameters of the fundamental orthogonal (sine polarized) mode was verified to satisfy the unitary relation-

ship. The analysis of this discontinuity including both the polarizations has been used to design a differential phase shifter as required in a polarizer application. A three section Chebychev transformer for the cosine polarized mode was first designed using the fundamental mode theory. The response of the initial design was optimized for a good return loss performance of both the polarizations and  $90^\circ$  phase shift between them at midband by altering the length of each ridged section. The number of modes used in the optimization was 20  $TE$  and  $TM$  modes of each polarization and the final analysis was performed using 40  $TE$  and  $TM$  modes of each polarization as the discontinuity is symmetric. The return loss and phase response of this type of a polarizer are shown in Figures 6 and 7.

## IV Conclusions

This paper describes a mode matching analysis of asymmetric discontinuities for dual mode filters and polarizers. Also, a differential phase shifter for a polarizer has been designed and optimized.

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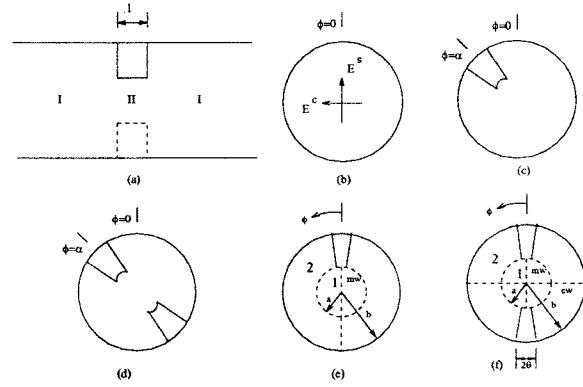


Figure 1: (a)-(d) Discontinuity regions I & II (e) & (f) Subregions 1 & 2 of II

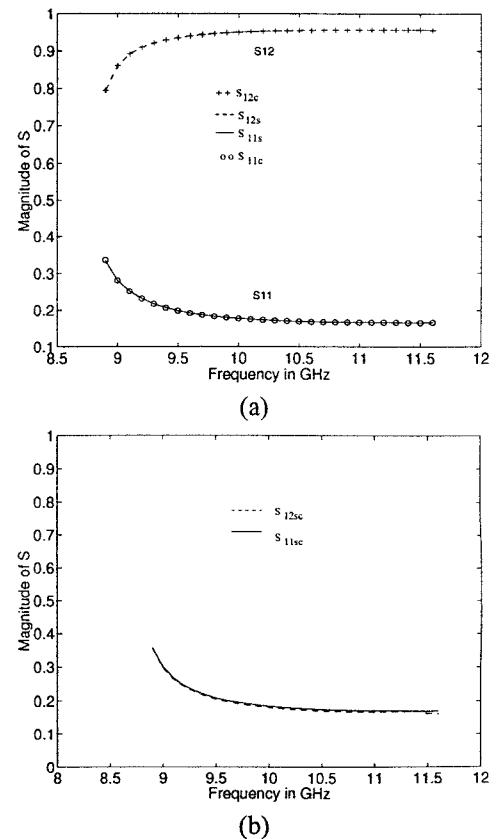


Figure 2: (a) and (b) Magnitude of S-parameters of a discontinuity from circular waveguide to single ridged circular waveguide of finite length,  $b=1.0\text{cm}$ ,  $a=0.4\text{cm}$ ,  $\alpha = 45^\circ$ ,  $\theta = 5^\circ$ ,  $l=1.0\text{mm}$

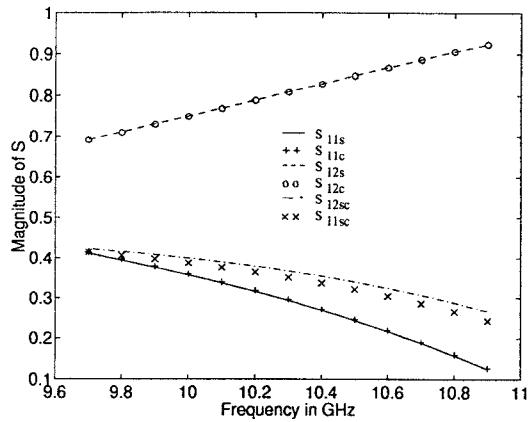


Figure 3: Magnitude of S-parameters of a discontinuity from circular waveguide to double ridged circular waveguide of finite length,  $b=1\text{cm}$ ,  $a=0.5\text{cm}$ ,  $\alpha = 45\text{deg}$ ,  $\theta = 10\text{deg}$ ,  $l=1.0\text{mm}$

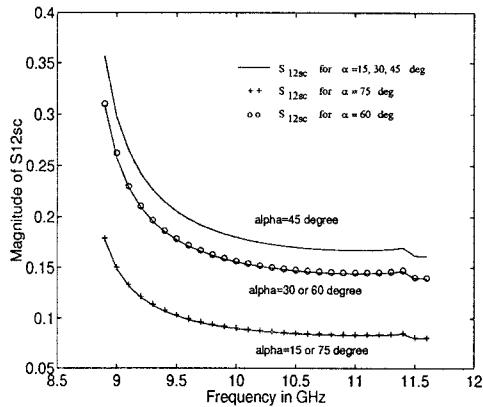


Figure 4: Magnitude of S-parameters of a discontinuity from circular waveguide to single ridged circular waveguide of finite length,  $b=1.0\text{cm}$ ,  $a=0.4\text{cm}$ ,  $\theta = 5\text{deg}$ ,  $l=1.0\text{mm}$

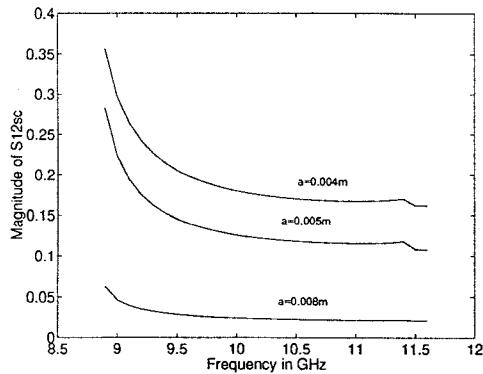


Figure 5: Magnitude of S-parameters of a discontinuity from circular waveguide to single ridged circular waveguide of finite length,  $b=1.0\text{cm}$ ,  $\alpha = 45\text{deg}$ ,  $\theta = 5\text{deg}$ ,  $l=1.0\text{mm}$ , solid lines -  $S_{12sc}$  for various values of 'a'

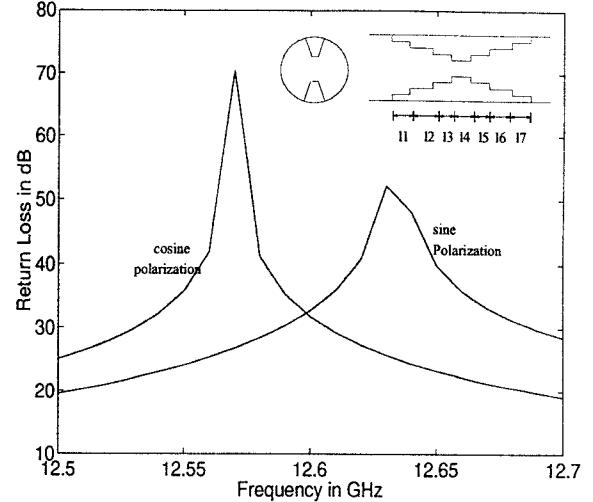


Figure 6: Return Loss of sine and cosine polarizations of the differential phase shifter unit of a polarizer Dimension in cm, section 1 and section 7:  $a=0.9$ ,  $b=1.0$ ,  $11=1.338$ , section 2 and section 6:  $a=0.7$   $b=1.0$ ,  $12=1.025$ , section 3 and section 5:  $a=0.56$ ,  $b=1.0$ ,  $13=1.118$ , section 4:  $a=0.5$ ,  $b=1.0$ ,  $14=3.680$

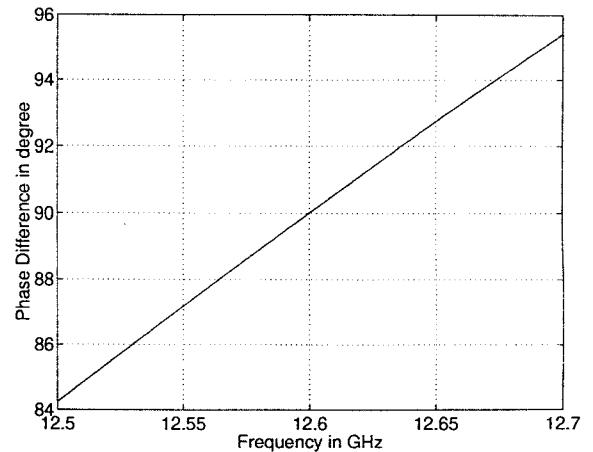


Figure 7: Phase difference between the sine and cosine polarizations of the differential phase shifter unit of the above polarizer